Mass-spring oscillations

www.lepla.eu

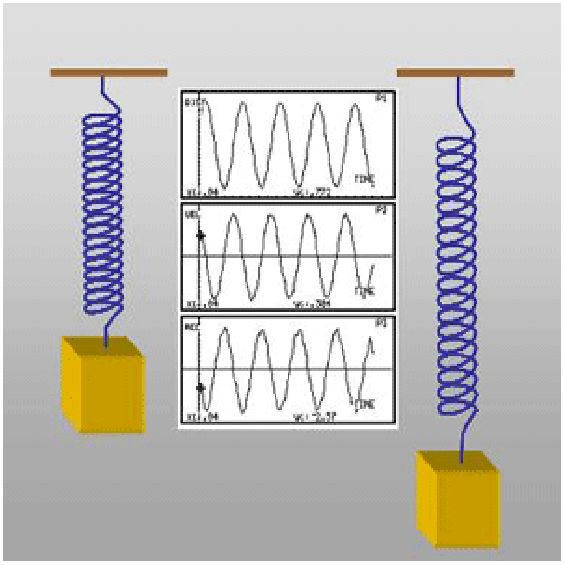
SOCRATES MINERVA Programme
99843-CP-1-2002-1-PL

Objective:

The experiment is designed to provide information on the behavior of a body hanging from a spring. The idea is to investigate simple harmonic oscillatory motion, observing how position, velocity and acceleration develop in time, how potential energy (elastic or gravitational) may be transformed into kinetic energy, or vice versa. Last but not least this investigation can offer the opportunity for using a Real Time data acquisition with a motion detector (SONAR).

You will be guided to perform a basic experiment on the oscillations of a body hanging from a spring and to analyze the experimental data. This will provide you with the basic facts and concepts about the phenomenon.
More experiments can be performed in order to investigate how the oscillation frequency is affected by the choice of the various system components:
- the type of spring (Hooke's law)
- the mass of the body
- the volume of the body
- the arrangement of a set of springs (series or parallel)

Data sample
Data analysis (TI)

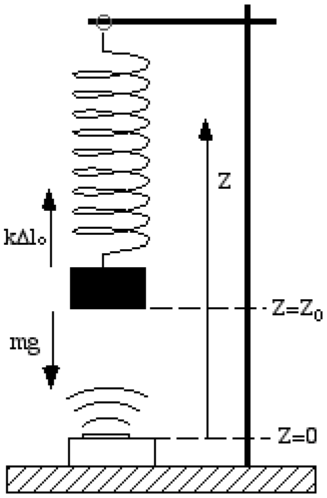


A deeper understanding of harmonic motion can be gained by comparing the behaviour of the mass-spring system with that of other oscillating systems.
Some of them are also illustrated in the LEPLA materials.

Contents

Theoretical model

Let us make some predictions on the characteristics of the motion of mass M attached to a spring, with elastic constant k, hanging from a fixed stand.



What is the elastic constant? See Hooke's Law (appendix 1)

At equilibrium the body is subject to two forces of equal intensity and opposite direction: the gravity force

$$P = M g \quad (1)$$

and the spring restoring force

$$F_0 = k \,\Delta L_0 \quad (2)$$

where $M, g, \Delta L_0$ are the mass, the gravity acceleration and the spring deformation when the body is at rest so that

$$\Delta L_0 = M g/k \quad (3)$$

When the body oscillates at the generic height z the force due to the spring is

$$F = P + E_{el} = Mg - k\Delta L \quad (4)$$

The motion equation is given by the Newton Law:

$$Ma = F \quad (5)$$

If we choose the origin of the reference frame in the equilibrium position and put

$$\Delta L = \Delta L_0 + z \quad (6)$$

by assuming that the motion is only in the z direction, the motion equation becomes

$$Ma = Mg \;\; k(\Delta L_0 + z) \quad (7)$$

This can be written in a form that does not depend on the gravitational field. Given the equilibrium condition

$$Mg = k\Delta L_0 \quad (8)$$

we get:

$$Mg = - kz \quad (9)$$

The equation is satisfied by the following solution

$$z = A_0 \cos(\omega t + \phi) \quad (10)$$
$$v = - A_0 \omega \sin(\omega t + \phi) \quad (11)$$
$$a = - A_0 \omega^2 \cos(\omega t + \phi) \quad (12)$$

where A_0 is the amplitude, ω is the angular frequency and ϕ is the phase, that depends from the position of the mass at $t = 0$
The motion is therefore periodic and the period T (i.e. the time required for one oscillation) is:

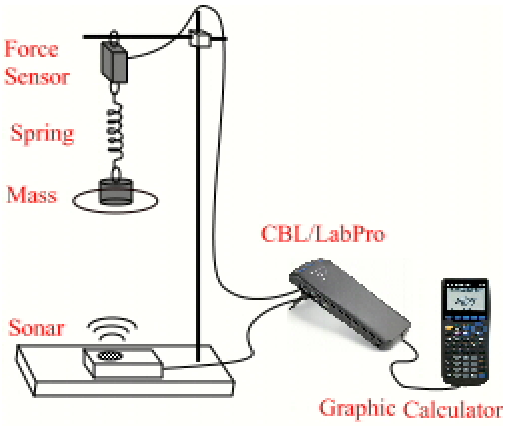
$$T = 2\pi \sqrt{M / k} \qquad (13)$$

On the ground of the theoretical model developed below, how would you expect the shape of the following plots to be?
- Position vs. time
- Velocity vs. time
- Acceleration vs. time
- Acceleration vs. position
- Velocity vs. position

Apparatus for mass-spring oscillations

To carry out the suggested experiments you should get the following equipment :

- Stand with hook
- Springs (0.5<k<3 N/m)
- Masses (20g to 200g) and mass-holder plate
- Motion detector (sonar)
- Graphic calculator
  (TI-83 or TI-89 /TI92+/Voyage200)
- CBL or LabPro interfaces
- Force sensor
  (to be used only in some experiments)



Qualitative observations

Before you start collecting experimental data you should observe the behaviour of the system, in order to decide what kind of measures might be most appropriate to investigate this kind of motion.

Hang a mass from the spring, then add one more.
What does change?
What do you expect will happen if you hang one more?
Why do you think so?
What would happen if the spring were "harder"?
What do you think are the relevant variables to describe the phenomenon?
(on web site or CD you can click for an answer)
What kind of plot would you choose to investigate the behaviour of the spring?
(on web site or CD you can click for an answer) Hang a mass from the spring and let it oscillate.
Is the motion "regular"?
What does it seem to remain constant from one oscillation to the other?
(on web site or CD you can click for an answer)
Which variables would you plot versus time to understand more about oscillations ?
(on web site or CD you can click for an answer)

Data acquisition
(TI 89/TI92+/Voyage 200, program Physics)

If you are not familiar with portable RTL systems you can see on web site or CDROM: Help box "portable RTL system"

We need to set the apparatus in order to investigate how the relevant variables change in time
To set up the data acquisition system and select the data acquisition mode, you can act as follows:

Connect the sonar to the data acquisition system, and launch the application PHYSICS,
and after the splash screen press ENTER key to access the MAIN MENU.

On MAIN MENU select 1: SET UP PROBES;
on menu NUMBER OF PROBES select 1: ONE;
on menu SELECT PROBE select 1: MOTION .
To measure displacements from equilibrium position:
on MAIN MENU select 5: ZERO PROBES,
and, on "choose channel" select MOTION
At the request to press the + key to set zero, verify first that the plate is steady at rest.

This procedure makes the distance to be measured in a reference frame whose origin is in the equilibrium position.
Therefore the body oscillations are positive when the plate rises (shortened spring) and negative when the plate falls (stretched spring).

Collecting data
On MAIN MENU select 2: COLLECT DATA.
Sample Time and Number of Samples values must be chosen according to the used spring constant and body mass.
Choose for example: SAMPLE TIME= 0,1 s and NUMBER OF SAMPLES= 30 (for a short acquisition useful in weakly damped oscillation) or SAMPLE TIME= 0,1 s and NUMBER OF SAMPLES= 150(to explore the behaviour of a damped oscillation)

Your system is now ready for collecting data.
Gently displace the plate upward (2-3 cm), and leave it free.
Start the data acquisition by pressing ENTER.
You will obtain three graphs for distance, velocity and acceleration as functions of time. (see Plot Analysis in Analysis Section).

If you are satisfied with the collected data you should save them by selecting SAVE from the MAIN MENU and the option "else", and then typing a file-name of 3 letters at least.

Data sample (TI 89/92+/Voyage200)

On web site or CDROM you can see a movie showing a mass-spring oscillator investigated with sonar , CBL and TI-89 and download data in various formats :

- Graphic Calculator data files (created by TI 89)
- MS-Excel data file MASSspring.xls
- Graphical Analysis data file

Data analysis with TI 89/T92+/Voyage200 and PHYSICS software

Many information can be drawn from the collected data.
Below you find the list of suggested types of analysis .
The analysis here documented was performed within the PHYSICS software, but alternative techniques may obviously be used (DataMate, Data/MatrixEditor, Graphical Analysis, Excel, ….).

- Plots analysis
- How to measure the Period
- Acceleration vs. displacement
- An alternative methods to evaluate the period
- Velocity vs. displacement
- Energy balance
- Damping

Plots analysis

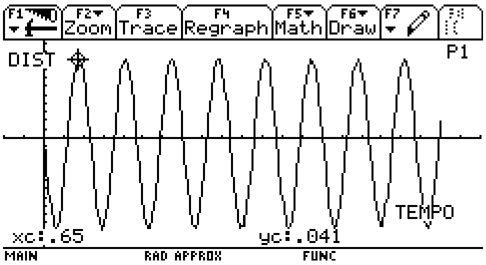


Figure 1: distance vs. time

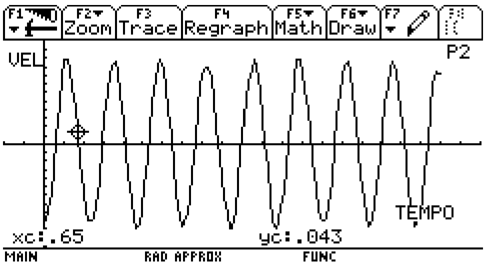


Figure 2: velocity vs. time

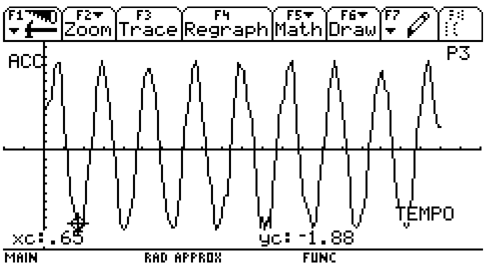


Figure 3: acceleration vs. time

Values of the data on the graphs can be read by using the TRACE mode: by pressing the (right/left) arrow keys you move the cross cursor along the curves and the corresponding values of the abscissa (xc) and of the ordinate (yc) appear at the screen bottom.

A few questions to be answered while analyzing the graphs can help you recognize the main features of the phenomenon.

1. How large is the oscillation amplitude A (i.e. the maximum elongation)?
2. How long is the time interval (period) after which the motion repeats virtually identical?
3. How does change in time the spring length ?
4. Why is the velocity positive in the time interval (t0; t2) and negative in the time interval (t2; t4)
5. When is the absolute value of the velocity maximum? And when is it minimum?
6. Why are the acceleration and the displacement always opposite in phase?

You can find the answer on LEPLA web site or on CDROM.

How to measure the Period

Look at the plots you got for position, velocity and acceleration versus time.

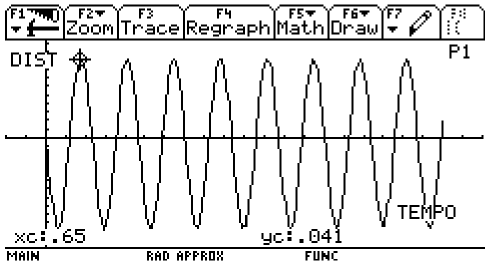


Figure 1: distance vs. time

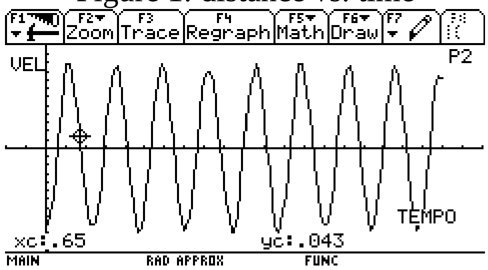


Figure 2: velocity vs. time

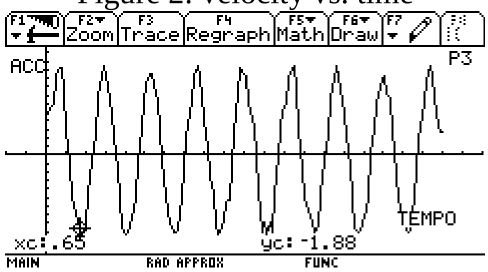


Figure 3: acceleration vs. time

The shape of the graphs is sinusoidal and using the cursor you can estimate the value of the period of oscillation.
To obtain a more accurate measure of the period choose the first and the last maxima (or minima) and calculate the time interval between the two.
The period can be calculated by dividing the time interval by the corresponding number of oscillations.
Is the period the same from all three plots? Why?
Why is it advisable to calculate the time interval between two peaks?
You can compare the experimental value of the period with that expected from theory
Are the two values compatible? If not, what could be the cause, in your opinion?

Acceleration vs. displacement

Look at the two plots of displacement and acceleration vs. time.

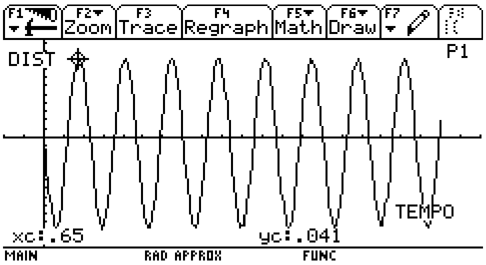


Figure 1: distance vs. time

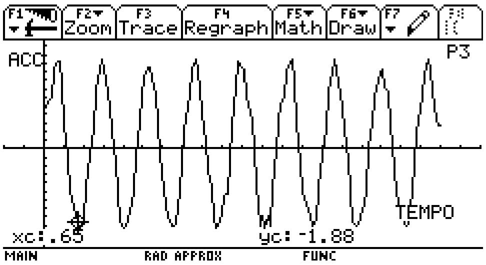


Figure 3: acceleration vs. time

To compare them it may be useful to put them one beneath the other.
You can notice that the period is the same, but phases are in opposition.
We can then suppose that:

$$a(t)= -const\ x(t) \qquad (1)$$

To check this hypothesis you can plot acceleration vs. displacement.
What kind of graph do you expect?

For a short description of the fitting procedure within PHYSICS, (see Help Box : Interpolate on web site)

By inspecting the plot we may detect the proportionality relation *a(t)=-const*x(t)* between *a* and *x*.
You can draw a straight line through your data, with negative slope *C*.
Which is the physical meaning of the proportionality constant *C* ?

From the plot of acceleration versus displacement we find

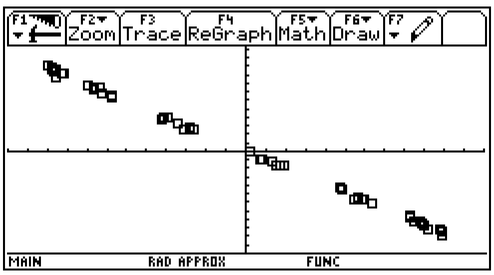


Figure 4: plot of acceleration vs displacement

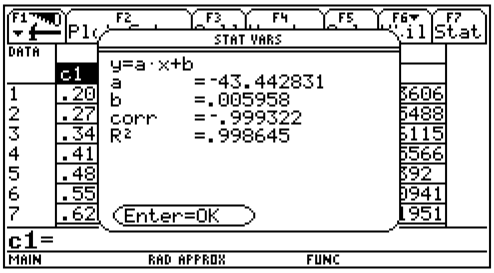


Figure 5 best fitting parameters

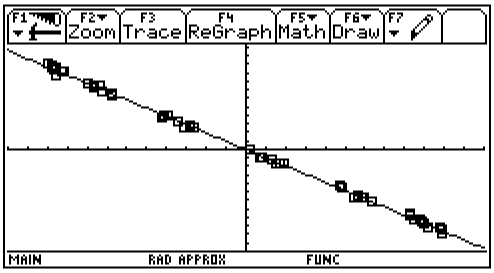


Figure 6 Best line fitting the data

An alternative methods to evaluate the period

The Newton's second law $ma=F_e$
with the definition of the elastic force $F_e= - k$ x (see theoretical model) gives:
$ma = - k x$ and therefore the acceleration is $a = -(k/m)\, x$

This gives to the slope C the meaning of ratio of elastic constant $k$to mass m

But the relation $a = - \omega^2 x$
tells us that the slope C of this plot also equals the square of the angular frequency $\omega^2=(2\pi/T)^2$
This gives us a measurement of the period as $T = 2\pi/(C)^{1/2}$.

Velocity vs. displacement

A less usual graphical representation of the motion may be obtained by plotting the velocity as a function of the position.
This is a phase-plot. What do you obtain ? Is it a closed curve or a spiraling one?

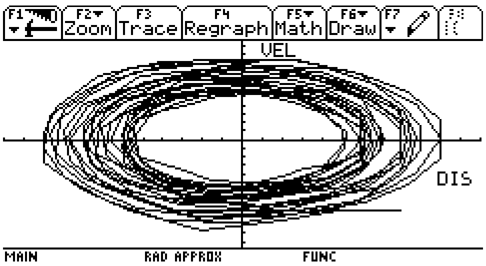


Figure 7: velocity as a function of the position.

The plot shows the effect of friction:
Without damping we get a closed ellipsis.
With damping the curve is a spiraling one

Energy balance

The net force acting on the body may be written $\boldsymbol{F} = -\boldsymbol{k}\,\boldsymbol{x}$, if the variable $\boldsymbol{x}$ measures the displacement from equilibrium position. This allows to calculate the energy balance where only two terms appear: the kinetic energy and the elastic potential energy (which, within this reference frame takes into account also the gravitational potential energy)

For a displacement $\boldsymbol{x}$ from the equilibrium position it performs the work:

$$W = \int F(x)\,dx = -\frac{1}{2}kx^2$$

This work is negative, because the force and the displacement are always opposite in sign, and it corresponds to an elastic potential energy $\boldsymbol{E_e}$ added to the system when the body moves from the equilibrium position.

Also the kinetic energy

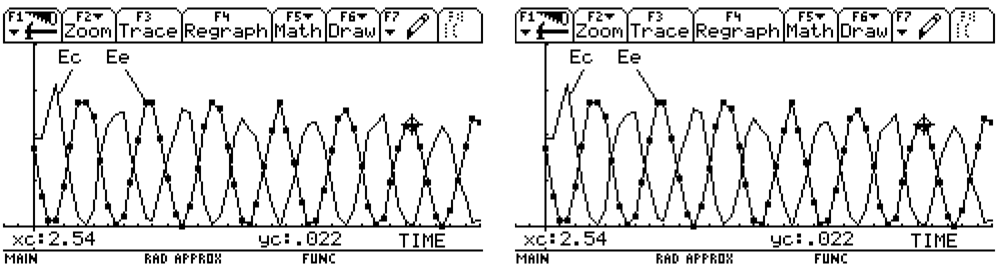
$$E_c = \frac{m}{2}v^2$$

changes during the oscillation , but the total energy $\boldsymbol{E}$ is conserved (on time intervals within which the dissipation may be neglected): i.e. the sum $\boldsymbol{E = E_e + E_c}$ must be a constant.

From the experimental data we may build the graphs of the two kinds of energy and of their sum as function of time (using for $\boldsymbol{E_e}$ the calculated value of the elastic constant k , or the measured force values *F(x)*, if we used also the force sensor).

These calculations may be performed using the DATA/MATRIX Editor, by opening the experimental Data-files, creating three new columns corresponding to the two energies and to their sum, and by plotting these values as funtion of time.

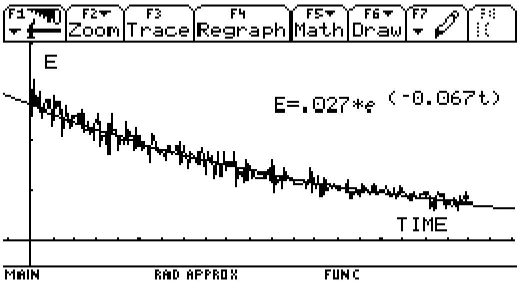
An example of these graphs , obtained with a slightly damped oscillator, is reported hereafter.



Ec=Kinetic energy and Ee =elastic potential energy, and their sum Ec+Ee, versus time.

Damping

By collecting data for a time interval long enough it is possible to study the total energy decay process. An exponential fit of the total energy as a function of time provides an evaluation of the decay time constant (see following graph)



By assuming that the dissipative force is substantially proportional to the velocity (through a proportionality constant α), the motion equation becomes:

$$m\frac{d^2x}{dt^2} = -2kx - \alpha\frac{dx}{dt} \qquad (1)$$

and letting $m\frac{d^2x}{dt^2} = -2kx - \alpha\frac{dx}{dt}$ we obtain

$$\frac{d^2x}{dt^2} + -2\delta\frac{dx}{dt} + \omega_0^2 x = 0 \qquad (2)$$

This equation , when the damping coefficient δ is small ($\delta \; \omega_o$) , has the solution:

$$x = Ao\exp(-2\delta t) - \cos(\omega t + \phi) \qquad (3)$$

where $\omega = \sqrt{\omega_0^2 - \delta^2} \approx \omega_0$ (3a)

Function (3) is a damped oscillation with amplitude A and phase φ) (the phase value depends on the value assumed for the origin of time scale).

Appendix 1: Hooke's Law

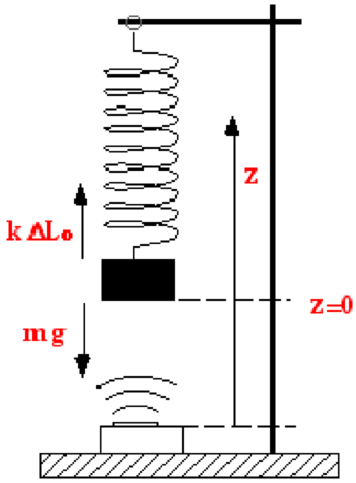


Figure1

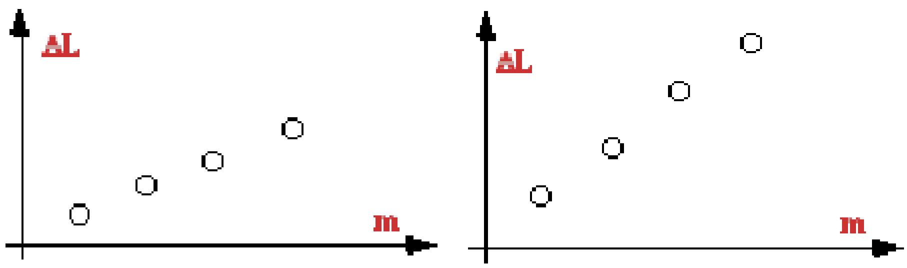
You see that different masses hanging from the same spring produce different changes in the spring length L. We will study the relation between m and ΔL

What does happen when we hang a mass to a spring?
We apply to the spring a force $F_w = mg$ directed downward (the weight of the mass).
When the mass rests in equilibrium the spring must produce a balancing force directed upward (we call it elastic force F_e) $F_w = - F_e$
To study the relation between mg and ΔL is equivalent to study the the relation between F_e and ΔL (there is only a change of sign)

We hang a mass m to a spring ,using a sonar to measure the changes of the spring length..
We will study the relation between m and ΔL

By repeating the measurement with *different masses* we obtain several pairs of values that may be traced on a plot. We may observe that the experimental points lay on a straight line.

By repeating the experiment with a *different spring* we obtain a similar plot where only the slope is different.



The slope of the fitting line depends only on the type of spring.
A spring is normally characterized by the force needed to produce unitary change in its length, a parameter that is named "elastic constant k" of the spring. In other words: $k = \Delta L/m$.