

Ball bounce



Experimental setup and procedure

Objective

In this experiment our objective is to study the motion of a bouncing ball using a **CBR detector** (calculator based ranger). We will measure the distance between the detector and the bouncing ball.

Materials

Ball, Tripod, CBR and TI-83 or TI-84

Procedure

- Before setting up the experiment make sure that you have the programs [FALL](#) and [CLEAN](#) in your TI-83. If not download them. Short instruction on how to download [here](#).
- A CBR is attached at the top of a high tripod. The CBR should be about 2 m above the floor. Connect the CBR and the calculator.
- Start the program FALL on your calculator. Choose 20 or 25 points per second. When the screen message is PRESS ENTER, hold the ball about 0,5 m below the CBR. Then press enter and when the CBR starts ticking drop the ball and let it bounce until the CBR stops ticking. If the ball bounces out sideways you may have to repeat the experiment.
- After the experiment the s-t-graph will be shown on the calculator. The y-axis is the distance from the CBR in meters and the x-axis is the time in seconds. Distance data are stored in list L_2 and time data in list L_1 .

If you are unable to conduct the experiment there are files set up so that you can look at the experiment and analyse previously collected data. Your options are shown below:

- [Look at a short video of the experiment](#) (3 Mbytes)
- [Get data to your TI83 or TI83 Plus](#). Short instruction how to download.

Analysis

To go to the analysis section choose one of the options below:

- [Data analysis using TI83 or TI83 Plus](#) (your own or downloaded data)
 - [Data analysis using Excel](#)
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Ball bounce

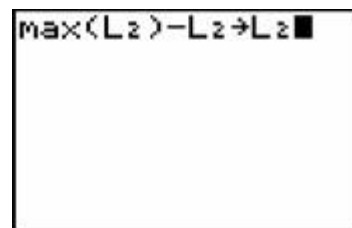


Data analysis (TI 83/TI84)

If you have any problems handling the graphing calculator help is available using the links that are underlined.

- To start with try to understand where the ball is at the different parts of the graph. Remember that the distances are measured from the CBR.
- To make it easier to analyse we will change the graph so that the y-axis shows the distance from the floor instead of the distance from the CBR.

To do this we use the fact that the top of the graph shown is when the ball bounces on the floor. To change the view we just take the maximum value in list L_2 and subtract all the values in L_2 from this. This is easily done by writing [max\(L2\)-L2→L2](#) on the home screen of the calculator. To look at the new graph press ZOOM ZoomStat on the calculator.



Analysis I: Mechanical energy

We look at the mechanical energy by calculating the potential and kinetic energy of the ball at different times.

The mass of the ball is 313 g.

- Determine the velocities in two different points before the first bounce. To calculate the velocity in a given point, choose a time interval symmetric with respect to the point. This is done by choosing data from the two adjacent points, the one before and after.

Now calculate the mean velocity in the time interval as $v = \frac{\Delta x}{\Delta t}$.

Approximate the velocity in the middle of the time interval with the mean velocity during the interval.

- Calculate the kinetic and potential energies in the two points where the velocities were determined. Perform the calculations in new columns in the spreadsheet. Finally calculate the total mechanical energy in these points.
- Repeat the calculations you made in above using two points between the first and second bounce. Choose one point when the ball is on its way upwards and one downwards.
- Continue the procedure above for points between the second and third, the third and forth bounce.
- Compare your results from the different calculations. Make statements about the energies of the ball in this experiment. Compare your assertions with what you can observe by just looking at the graph.

When you have finished your analysis you can compare it with the completed analysis

Analysis II: The height of each bounce

Another way to investigate the energy loss in each bounce is to study the maximum heights of consequent bounces. Since every maximum height gives information about the maximum potential energy we have to determine those heights. In each of these points the kinetic energy equals zero, because the velocities are zero.

To get this information, look at the graph showing distances from the floor.

- Use TRACE and find the height from which the ball was dropped. This height is placed as the first element in list L_5 . An easy way to do this is to use the fact that the x and y-values are accessible from the X and Y-registers on the calculator when you trace. In practice proceed like this: Trace until your data point is beneath the cursor. Press STAT, choose EDIT and place the cursor in $L_5(1)$ and press Y (ALPHA 1). The y-value will be pasted into $L_5(1)$. This correspond to the height from which the ball was dropped. Therefore put 0 in $L_4(1)$ stating that no bounce has yet occurred.
- Repeat tracing to the following maximum heights of as many bounces that you have got and put them into lists L_4 and L_5 .
- [Plot the height as a function of "bounce-number"](#), i.e. list L_4 as the x-list and list L_5 as the y-list.
- Try to fit a curve to these data using a adequate [regression](#).
- What conclusions can be make. After how many bounces will the ball stop bouncing ?

When you have finished your analysis you can compare it with the completed analysis

Analysis III: The movement between two bounces

In this section we study the motion between two bounces.

- [Return to the original graph \(L1 and L2\)](#)
- Use the [Select-command](#) to select the part between the first and second bounce. Paste the selected part in lists L_4 and L_5 .

Doing so prevents your recorded data to be erased. On the other hand those data that already are in the target lists, L_4 and L_5 , will disappear because they will be overwritten. If you do not want to delete the data from the previous analysis you have to [rename those lists](#). However, you do not need these data anymore so it's not necessary to do this.



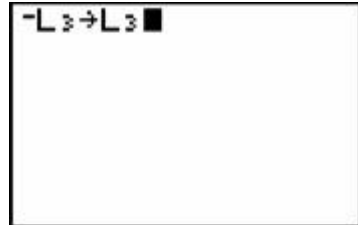
- Use ZoomStat to zoom in on the part that you have selected.
- Try to fit a curve to these data using a suitable [regression](#).
- Do the constants in the regression equation have any physical meaning. If so which ones and what do they mean ?
- What kind of motion is it? What does this tell us about the force acting on the ball between the bounces ?

When you have finished your analysis you can compare it with the completed analysis

Analysis IV: The velocity of the ball

In list L_3 the velocity of the ball is stored. These data are calculated using a numeric derivative of the original position-time-data.

This means that positive direction is downwards, from the CBR. To change this, perform the following transformation on list L_3 on the home screen: $-L_3 \rightarrow L_3$ (the arrow is STO). This changes the sign of all velocities. Now velocity data and distance data correspond. Before you make a velocity-time graph try to imagine what it would look like. Sketch it roughly by hand.



- [Plot the velocity-time-graph on the calculator \(L1, L3\)](#). Does it look like you have expected? If not, why ?
- What happens during the linear parts? Can you make a guess of their slopes? What happens with the ball when the velocity changes sign abruptly?
- [Select](#) one of the linear parts of the graph and try to [fit a straight line to it](#). Observe that the help section gives a quadratic. Is the slope what you had expected?
- Calculate the mean acceleration of the ball when it bounces the first time. Estimate the resulting mean force on the ball during the bounce. What forces are acting on the ball when it bounces?

When you have finished your analysis you can compare it with the completed analysis

Ball bounce

Mechanical energy: Completed analysis (TI 83/TI84)



First we choose a point on the way down. To calculate the kinetic energy in that point we need to find the velocity. To do this we use three consecutive points with “our” point in the middle. Then we calculate the mean velocity in the small time interval from the point before to the point after. This is a good approximation of the velocity in the point in the middle.



$$v = \frac{\Delta s}{\Delta t} = \frac{(0.9889 - 1.2060) \text{ m}}{(0.7188 - 0.6390) \text{ s}} = -2.72 \text{ m/s}$$

Where the negative sign tells us that the ball is going down.

Knowing the balls mass is 0.313 kg we can calculate the potential and kinetic energies in the point and hence the mechanical energy.

$$W_p = mgh = 0.313 \cdot 9.82 \cdot 1.1047 \text{ J} = 3.40 \text{ J}$$

$$W_k = \frac{mv^2}{2} = \frac{0.313 \cdot (-2.72)^2}{2} \text{ J} = 1.16 \text{ J}$$

$$W_{mek} = W_p + W_k = 3.40 \text{ J} + 1.16 \text{ J} = 4.46 \text{ J}$$

We repeat these calculations for one more point going down before the first bounce, one point going up again, another going down and so on. The result for some of the points is gathered in the table below.

t / s	v / m/s	W_p / J	W_k / J	W_{mek} / J
0.679	-2.72	3.40	1.16	4.46
0.879	-4.62	1.15	3.34	4.49
1.078	3.39	1.50	1.80	3.30
1.677	-2.45	2.39	0.94	3.33
1.997	3.01	1.04	1.42	2.46
2.476	-1.64	2.07	0.42	2.49
2.835	2.36	1.07	0.87	1.94
3.274	-1.91	1.38	0.57	1.95

From the table above it is obvious that the mechanical energy is the same between bounces and that mechanical energy is lost when the ball bounces. That the energy is conserved between bounces imply that effects such as air resistance can be neglected within this experimental accuracy.

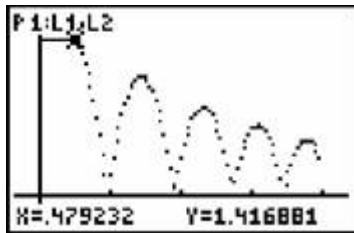
The energy losses when the ball bounces depends on heat losses during the deformation of the ball. Part of the balls mechanical energy is transferred into heat. As a result of this the temperature of the ball and the floor becomes slightly higher.

Ball bounce

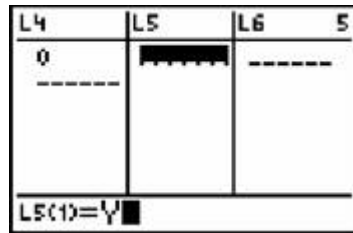


Bounce heights: Completed analysis ((TI 83/TI84))

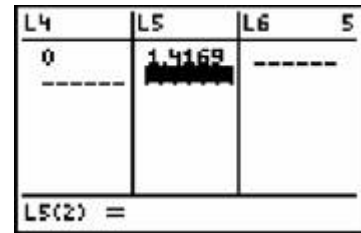
The heights of the bounces is pasted into list L_5 as described in the analysis. The pictures below shows the procedure step by step.



Position the cursor at the plateau before the ball is dropped.

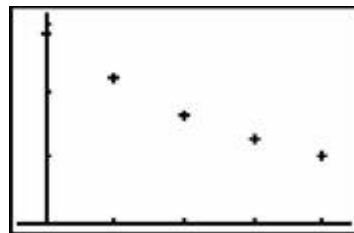


Press Stat, edit to get to the list editor. Input 0 in $L_4(1)$ and Y (alpha 1) in $L_5(1)$.

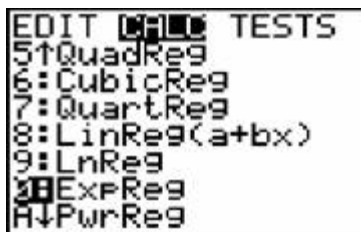


The value from the Y-register will be pasted into $L_5(1)$.

When we have all the data in the lists we [graph the height as a function of bounce number](#). The scatter plot looks like this:



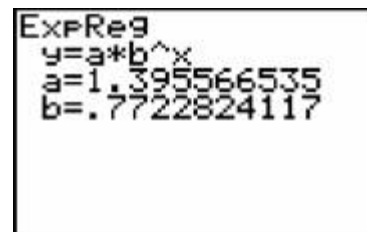
A reasonable curve fit seems to be an exponential function. The steps below takes you through how to make an exponential regression.



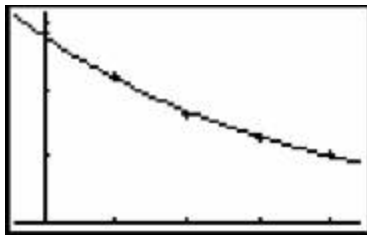
Press STAT. Chose CALC and go down to ExpReg. When you press ENTER the command will be pasted to your home screen.



The syntax is L_4, L_5, Y_1 for x and y lists and Y_1 . Y_1 is found under VARS, Y-VARS, Function and the purpose of it is to paste the regression equation to the function editor.



After the regression the constants are shown on the home screen.



Press ZOOM ZoomStat to show the graph with the regression line. If you want the x-axis to show go to WINDOW and set Ymin = 0.

As can be seen the exponential model fits nicely. The equation $y = 1.40 \cdot 0.772^x$ gives us the following information:

The initial height is 1.40 m.

After each subsequent bounce the ball reaches up to a maximum height that is 0.772 of the previous one.

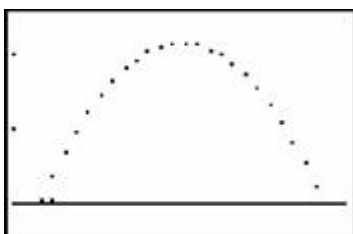
Since potential energy is linear with respect to height, $W_p = mgh$, this means that 77 % of the total mechanical energy is preserved in each bounce. Another way to say this is that the system loses 23 % of its mechanical energy during each bounce.

Ball bounce

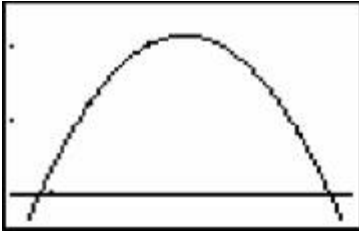
Motion between bounces: Completed analysis ((TI 83/TI84))



To analyse the motion between two bounces we first have to cut out that part of the graph. This is easily done with the command [Select](#), which is found either in LIST (2:nd STAT), OPS or in the CATALOG (2:nd 0). After the selection is done and the window settings have been adjusted the first bounce looks like this.



A reasonable curve fit seems to be a quadratic function. Therefore we perform a [quadratic regression](#) on the data. The constants and the curve fitted is shown below.



As can be seen the quadratic model fits almost perfect. In the equation

$y = -4.88x^2 + 13.9x - 8.86$ only one of the constants have any physical meaning. That is the -4.88 which is half the acceleration of the ball. If we double it we get -9.76 , which is very close to $g = 9.81 \text{ m/s}^2$. The negative sign tells us that the acceleration is directed downwards.

The quadratic model also tells us that the motion is uniformly accelerated, i.e. the net force on the ball is the same at all times. This force is the weight of the object and can be calculated by

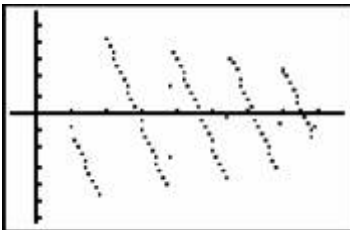
$$F = mg = 0.313 \cdot 9.82 \text{ N} = 3.1 \text{ N}$$

Ball bounce

The velocity of the ball: Completed analysis ((TI 83/TI84))



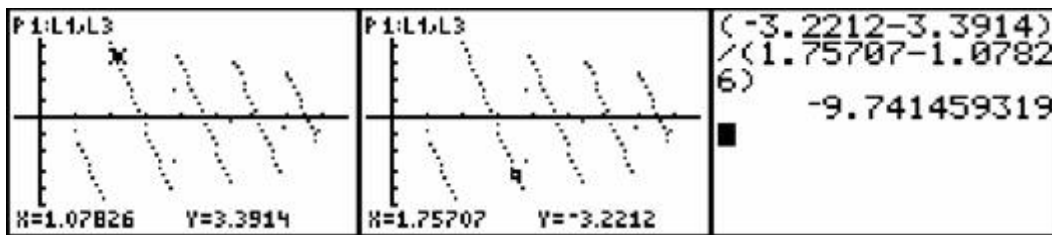
Following the steps in the analysis section we get the following velocity-time-graph. The velocities are measured in m/s and times in s.



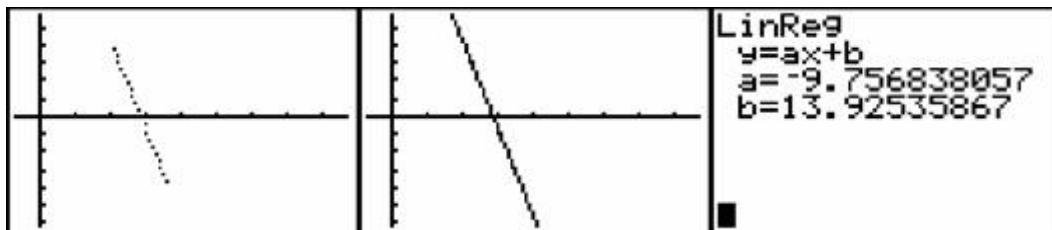
The linearity is because the acceleration is the same while the ball is in the air. This means that the velocity changes equally much during equal time intervals. When the ball bounces the velocity changes sign and the velocity changes are big during a short time due to the force from the floor. Note that every bounce gives the ball a slightly lower velocity when leaving the ground because mechanical energy is lost in each bounce. In the graph the intersection with the x-axis correspond to zero velocity, i.e. when the ball is at the highest point.

All the linear parts of the graph have the same slope. During this time the ball is in the air only affected by its own weight. The slope of the linear part therefore should equal the acceleration due to gravity. We can check that either by calculating the mean acceleration choosing two

points and calculating $a = \frac{\Delta v}{\Delta t}$

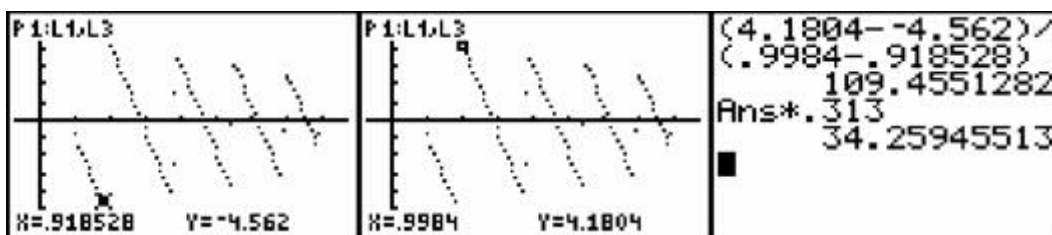


Another way of doing this is to [select](#) one of the linear parts and make a linear regression on it.



As we see the slope is very close to g .

To estimate the force acting on the ball when it bounces we try to estimate the time it is in contact with the ground. Using TRACE we find this time interval to be approximately 0.1 s. Using this data we find the mean acceleration to be -110 m/s^2 . Giving a net force of $F = ma = 0.313 \cdot -109 \text{ N} = -34 \text{ N}$



This is more than 10 times it's own weight. The peak force is roughly twice this value.
