Slit-Diffraction Spectra

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SOCRATES MINERVA Programme
99843-CP-1-2002-1-PL

The aim of the experiment is to explore the basic features of diffraction and interference of light.
For this purpose data about diffraction patterns are collected using an optical bench where position and light intensity are measured by two sensors interfaced to a graphing calculator.
Experimental data are then compared to theoretical patterns calculated following Fraunhofer model of diffraction.

Contents:

- Theoretical model
- Apparatus setup
- Data acquisition
- Data sample
- Data analysis (TI89)
- Complete Data Analysis (MSExcel)

Theoretical model

In order to make predictions on the behaviour of light intensity as a function of position in the diffraction spectrum, one must use a theoretical model of the phenomenon. Let us consider a slit of width a, divided into N parallel stripes of width Δx, illuminated by a light source producing a plane wavefront. Applying the Huygens principle we may assume each stripe to be a secondary source producing its perturbation on the point P, whose position on the screen is related to the angle θ

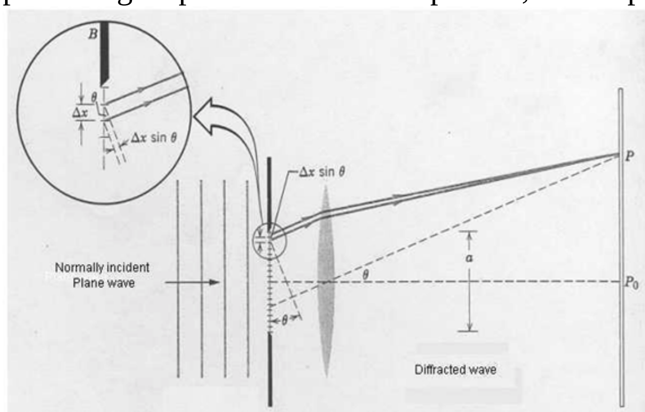


Figure 1
(from Halliday D., Resnick R. “*Fundamentals of physics*”, J Wiley & Sons, inc, 1990)

For not too large values of θ, we may assume that all the perturbations in P due to different stripes have the same amplitude E. To calculate the amplitude E_θ of the whole perturbation for each value of θ we represent the single perturbations ΔE_θ as rotating vectors, named phasors.
This approach gives the relative phase angle of each perturbation in terms of the phasor angle ϕ, and the intensity as phasor module. The total perturbation E_θ at each point P(θ) on the screen is obtained adding up the single perturbations so that the square of E_θ is proportional to the local light intensity I_θ.

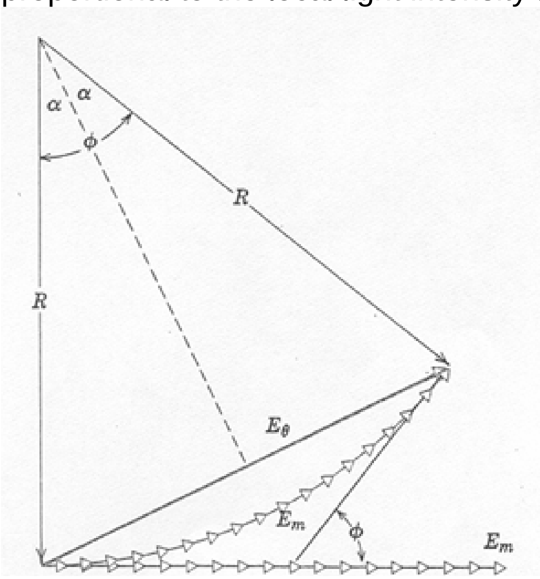


Figure 2
(from Halliday D., Resnick R. ³*Fundamentals of physics²,* J Wiley & Sons, inc, 1990)

When the number N of stripes goes to infinity, the stripe width dx goes to zero, and the polygonal made of phasors in figure 2 becomes an arc of radius R. At the centre of the diffraction spectrum all perturbations are in-phase, the arc becomes a straight segment of length E_m equal to E_θ.The angle ϕ represents the phase shift between the extreme phasors equal to that between the light beams coming from the slit borders.
From geometrical analysis of figure 2 we may write:

$$E_\theta = \frac{2R \cdot \sin\phi}{2} \cdot \phi$$

where ϕ is measured in radians :

$$\phi = \frac{E_m}{R}$$

Combining the above equations we get:

$$E_\theta = \frac{E_m}{\phi/2} \cdot \frac{\sin\phi}{2}$$

or

$$E_\theta = E_m \frac{\sin\alpha}{\alpha} \quad (1)$$

Where $\alpha = \phi/2$. Recalling the definition of ϕ as the phase difference between rays coming from the slit borders, and of the difference in the optical path between these rays is $a\sin\theta$ we obtain the relation:

$$\phi = (2\pi/\lambda)(a\sin\theta)$$

or

$$\alpha = \pi a \sin\theta/\lambda$$

Using equations (1) and (2) we obtain the perturbation amplitude as a function of the angle θ. The light intensity I_θ is proportional to the square of the amplitude:

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$$

The relative intensity (normalized to the intensity of the central peak I_m) is

$$I_\theta / I_m = [\sin(\pi a \sin\theta/\lambda]^2 / (\pi a \sin\theta/\lambda)^2$$

If the experimental setup is within the limit of Fraunhofer diffraction (paraxial wave equation or $x \ll D$) we may let $\sin\theta = x/D$ obtaining the simpler form:

$$I_\theta / I_m = [\sin(\pi a\, x/D\lambda]^2 / (\pi a\, x/D\lambda)^2$$

For *one single slit* the theory predicts that the relative intensity of the secondary peaks of index *i* to decrease as $I_i/I_o = 4/[\pi(2_i+1)]\ 2$, i.e. : $I_1/I_o = 4.5\%$, $I_2/I_o = 1.6\%$, $I_3/I_o = 0.8\%$.
The positions x_m for minimum intensity are solution of the equation $\sin(\pi a x/\lambda D)=0$,

or $\pi ax/\lambda D = m\pi$: therefore $x_m = m\lambda D/a$.

The distance between adiacent minima is $\Delta x = \lambda D/\alpha$. The predicted secondary peak positions are approximated by $x_m = 1.5\, m\lambda D/\alpha$.

In the following we report some simulations comparing different slit width (figure 3) , obtained using Excel.

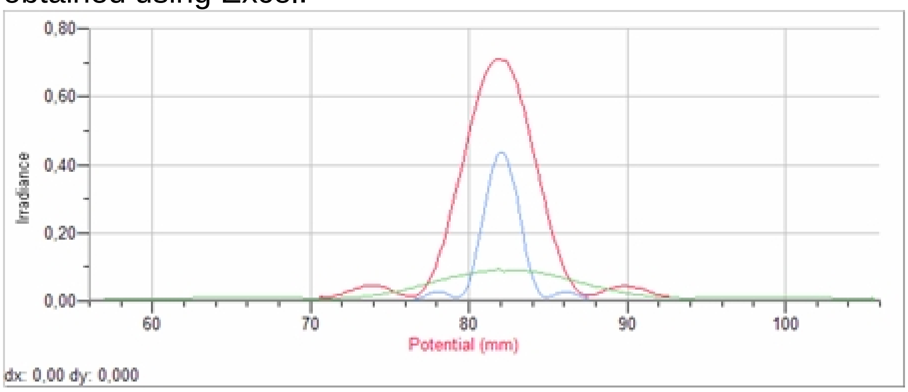


Figure 3
Spectra calculated for various slit-widths *:*
red a = 160 mm blue a = 80 mm green a = 40 mm

Predicted spectra for multiple slits

For *multiple slits* (pitch= p , width = a) in the same approximation $x << D$, and with the further approximation p / $D<<1$, *for* N slits the predicted pattern is described by the function:

$$I(x)/I_o = [\sin^2 (\pi ax/D\lambda)]/(\pi ax /D/\lambda)^2 [\sin^2(N\pi px/D\lambda)]/[N\sin(\pi px/D\lambda)]^2$$

In spectra of multiple slits the secondary peak intensity decreases as $I_i / I_o = [(Np)/(\pi ia)]^2$

$[\sin^2 (i\, \pi a/D)]$ approximated by $I_i / I_o = 2N^2\pi/ipa$, inversely proportional to index i with slope (for N =2) of about 2.55 p/a .

The minima positions x_m depend on the ratio l/a : $x_m = m l D/a$, while the peak positions X_m depend on the ratio l/p : $X_m = m\lambda D/p$.
Distance between adiacent peaks is $\Delta X = \lambda D/p$.

Examples of experimental data and comparison with calculated spectra

The following plots show the distribution of illumination (in arbitrary units) vs. position (in mm, relative to maximum position) under several experimental conditions. Plots on the left show experimental data (obtained with screen-slit distance D=720 mm), plots on the right show the calculated spectra.

Single slit

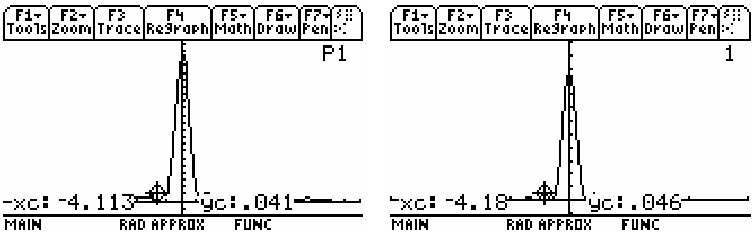


Fig. 4 -Experimental data (A) and simulation (B) for slit width a =160 mm,
In figure 4 distance between first and second maximum is **Dx=4.11** mm in the experimental data and **Dx=4.18** mm in the calculated spectrum: the two patterns agree within 2%.

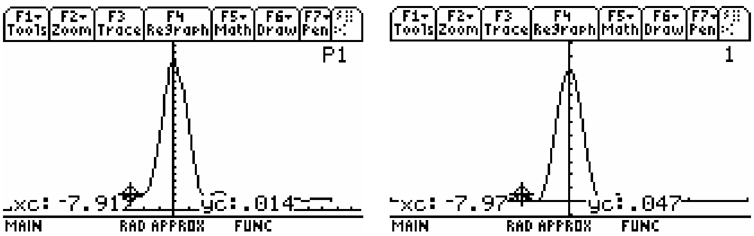


Fig. 5 -Experimental data (A) and simulation (B) for slit width ***a =80*** *mm,*

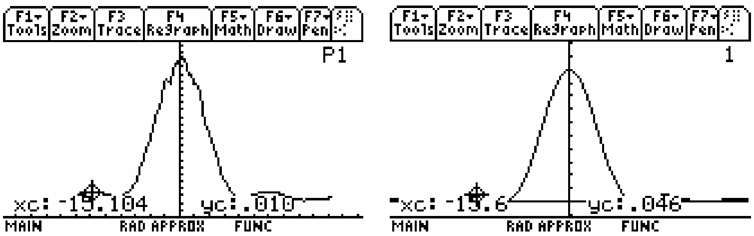


Fig. 6 -Experimental data (A) and simulation (B) for slit width ***a =40*** *mm*

Similar results can be obtained using **multiple slits**. Here we report some obtained with double slits,

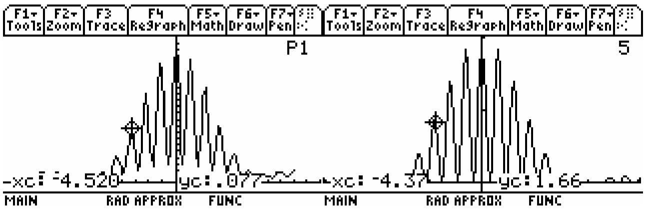


Fig. 7 -Measured (A) and Calculated (B) : width a=40 mm, slit separation p=250 mm

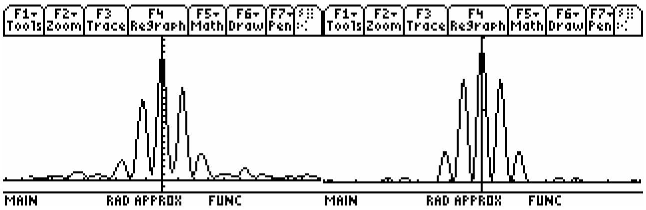


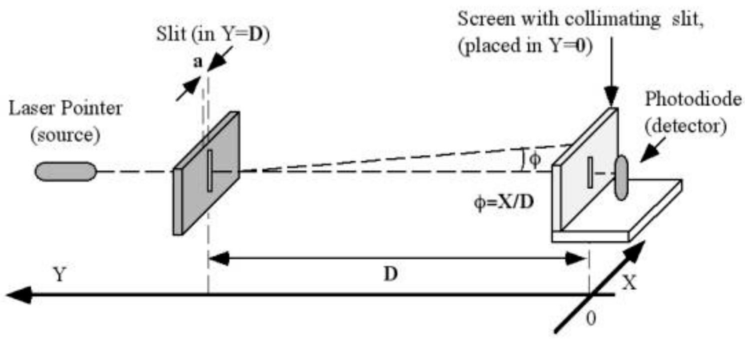
Fig. 8 -Measured (A) and Calculated (B) : width a=40 *mm*, slit separation p=125 *mm*

Apparatus setup

The apparatus is an optical bench (produced by LabTrek:http://www.labtrek.net), made of a "T" shaped aluminum rail, with a preamplified photodiode (coupled to a potentiometric position sensor) mounted on one end and a source of monochromatic light well collimated (a low cost laser-diode pointer) placed on the other end. In between source and detector a slit-holder provides an easy arrangement for PASCO slits.

- Besides the optical bench you need :
- TI-89 or TI83 or Voyage graphing calculator
- black connecting cable
- CBL or LabPro interface
  PASCO OS8523 kit: Single-slit Disk -- 4 single slits , (a=0.16 mm, a= 0.08mm, a=0.04mm, a=0.02 mm) variable-width slit (0.02-0.20 mm), Multiple-slit Disk -- 4 double slits (a=0.08 mm d=0.25mm , a=0.08 mm d=0.50mm , a=0.08 mm d= 0.25mm), 4 multiple slits (2, 3, 4 or 5 slits), 4 single/double-slit comparisons, variable double-slit (slit separation 0.125-0.75 mm)

The experiment consists essentially of three steps:
- prepare the diffraction pattern on the screen by properly adjusting the components on the optical bench,
- setup the data acquisition system,
- acquire data and properly graph them on the calculator display.



The light sensor is swept along the X direction by rotating a hand-wheel and the light intensity is recorded at the chosen constant rate together with the sensor position. Normally 100 points are more than enough to obtain a good record at 20 Hz rate in half a minute. Before starting the experiment you must be shure to have loaded the software SCIENCE (see the list of files PRG within VAR-LINK). To enter VAR-LINK press yellow key and then (-) key.

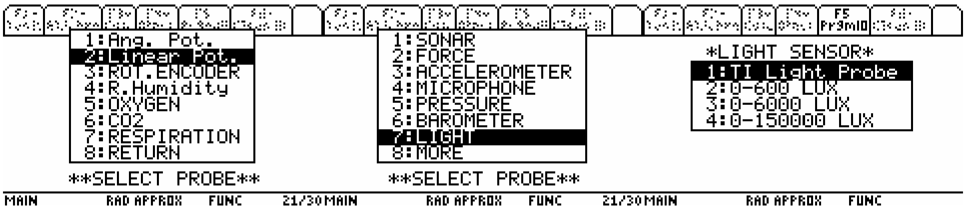
Now proceed as follows:

- connect the position sensor cable to CHANNEL 1 and the light sensor cable to CHANNEL 2
- rotate the handle of the position sensor until the slit on the screen is aligned with the rail (use the rule attached to the sensor base for precise alignment).
- Use the movable screen (with a rubber band horizontally wound around it as a marker) to check the horizontality of the laser beam: the spot must stay at the same height on the screen when you slide the screen along the rail.
- place the laser diode on the rail at the end opposite to the sensors and connect it to the current regulator using the cable with two jack plugs. Connect the current regulator to 220V main line and turn the knob fully clockwise for maximum brightness of the laser beam. (avoid to point the laser beam toward a person's eye: it may seriously damage the retina)
- adjust the position of the laser diode holder until the laser spot height is on the horizontal line on the screen and centered on the screen collimating slit.
- place one of PASCO Slit Disk on the holder, and fix the screw to assure a proper angular orientation of the Slit Disk on the holder.
- Annotate the chosen value of the slit- screen distance D
- choose the slit pattern you want to use by turning the Disk until one of the slot at the disk border fits to the stopper : the chosen slt pattern will be aligned on the laser beam.
- Be sure that the resulting pattern on the screen (a series of dots aligned on a line) is horizontal: if not the Slit Disk
- Prepare the graphing calculator for data acquisition fron the two channel of CBL or LabPro.

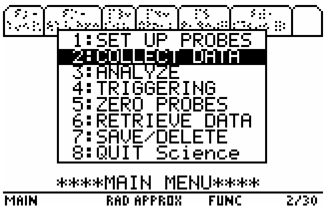
Data Acquisition

Detailed informations on the use of the graphing calculator for data acquisition may be found in “ Introduction to the hand held technology (TI-89 and CBL) ” .Here you find a few suggestion on using the software SCIENCE with TI-89, TI92+ or TI-Voyage.

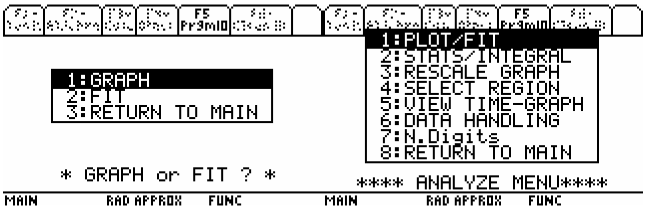
We suggest to choose a single slit in the first trial of this experiment.
Run the software SCIENCE . To instruct the interface about the sensor choice on the MAIN MENU select “1: SETUP PROBES” and choose “2: TWO”. Then look for the desired kind of probes on the list: on the third window you find “2: Linear Pot.” that is the proper choice for the displacement sensor calibrated for a span of 10 cm, and on the first window choose “7: LIGHT” for the light sensor, and then “1: TI light probe”.



From MAIN MENU chose “2: COLLECT DATA” and then “2: TIME GRAPH” , in order to make an acquisition of many values of the sensor position and of the light intensity, at constant rate, while moving the sensor to scan the diffraction spectrum.



To choose the rate for data acquisition enter the time interval between two acquisition (for example 0.2 seconds) and the number of samples to be collected (e.g. 100).
At this point you press ENTER and you start turning the handle to move the light sensor across the light pattern in order to scan the diffraction spectrum.
The more constant is the movement the more accurate will be your measurement.
After completing a data collection, the measured values will be stored into two lists as function of time (L1= time, CH1=L2=position, CH2=L3= light intensity).



You may view the light intensity vs. time plot, using the option “5: VIEW TIME GRAPH” in “ANALYZE MENU”, in order to check the result of your acquisition, but you must remember that this plot is not a diffraction spectrum: you must plot the light intensity vs. position.
The software SCIENCE allows to access to the stored data taken as time plots, or as plots of the light intensity (CH2) as a function of the position (CH1) .
From the MAIN MENU select ANALYSIS and then PLOT/FIT

To make the desired plot, select 1: GRAPH , and then select CHANNEL 1 for X axis and CHANNEL 2 for Y axis.
The graphing calculator will display the plot (hereafter is shown the case of single slit diffraction pattern).
This plot has for abscissa the absolute position of the light sensor. In order to transform the absolute position into displacement with respect to the central position you must modify the list 2 using the “6: DATA HANDLING” option in the “ANALYZE MENU”, choosing the option “5: LINEAR TRANSF.” .
In the next screen you choose the list to be modified (position is in CHANNEL 1)

As SHIFT FACTOR you choose the position of the central peak found in the previous plot, (with negative sign in order to subtract, in the example -35.7), and as SCALE FACTOR you leave the default value=1.

After pressing ENTER the list is modified and, you may repeat the procedure to get the plot of the spectrum with the shifted abscissa, i.e. the light intensity versus displacement with respect to the central peak.

Analyse the plot you obtained (diffraction pattern) and try to answer to the following questions:

- Can you distinguish the light intensity maxima and minima?
- Is the plot symmetric?
- Which?
- Now change the slit width and compare the new plot with the previous one:
- What is changed?

Data Sample

The data sample (a single slit diffraction spectrum, with slit width a=80 m m and slit-screen distance D=700 mm) may be downloaded from CD or web site in two format:

- File produced by graphing calculator TI-89: d80700.89c

This file was produced by the application SCIENCE. May be opened by the software TI-Connect or TI-GraphLink or TIConnect and tranferred from PC to graphing calculator using the TI-GraphLink cable (gray, black or USB)

- File exported for MS-Excel analysis: diffr.xls
- File with complete analysis with MS-Excel: diffrEN.xls.

Data Analysis (TI89)

We suggest you a procedure to analyse the experimental data obtained with a single slit. Detailed informations on the graphing calculator and on data handling may be found in the "Introduction to the hand held technology (TI-89 and CBL)".

a. Study of the diffraction spectrum plot

To display light intensity versus position you must build a plot with the measured sensor position values (CHANNEL 1) on x-axis and measured light intensity values (CHANNEL 2) on y-axis .
This plot is a diffraction spectrum.
Looking at the diffraction spectrum tyry to answer to the following questions :

- Can you detect the minimum values and the maximum values of light intensity ?
- Do you see the central peak?
- Do you see a symmetry in secondary peaks with respect to the central peak ?

If you took data also with a slit with different width, compare the different plots.

- Which are the differences between the two plots?

b. Comparison between experimental and simulated stpectra.

The recorded experimental spectrum may be compared with a simulated spectrum calculated through a theoretical model.
In order to make possible this comparison, the measured background light must be subtracted from the measured light intensity values, and the measured values of absolute position must be converted into values of position relative to the central peak (i.e. subtracting the central peak position value from all the measured position values).

Be careful with units of measurements:
e.g. the units of length used for the values of the light sensor displacements x must be the same as that used for the slit-screen distance D: if you used the default units (mm), also D must be entered in mm. Note that in the theoretical model some quantities always appear in ratios (x/D, a/ l); this allows the use of mixed units (e.g. x,D in mm and a, l in mm), provided that the quantities in the ratios are given in the same units.A fast check of compatibility of experimental and simulated plot is obtained by comparing the distance between the central peak and the first secondary maximum.

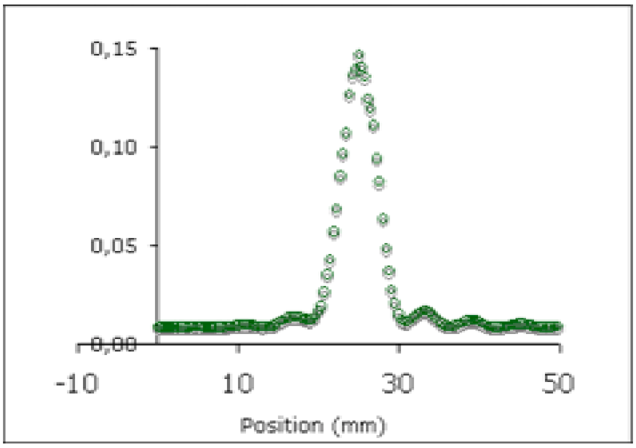
Your analysis may be compared with the example of "Complete Analysis made with Excel" in the next pages.
On the web is available also an exemple of complete Analysis made with TI89.

Complete Analysis (MS-Excel)

The experimental data were imported into Excel worksheet using the TI Graph Link cable connecting the graphing calculator with the PC.

You can downalaod diffr.xls from CD or web site.
Time values (in seconds) are in column A, position values (in cm) in column B, light intensity values (in the default arbitrary units) in column C.
The complete analysis here reported is for a single slit (width a = 80 µm and slit-screen distance D = 700 mm).

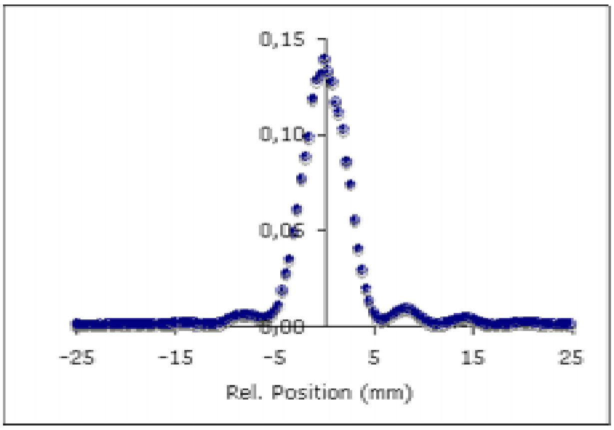
The diffraction spectrum is obtained by placing on x axis the position values (column A) and on y axis the light intensity values (column B).



To compare the measured spectrum with that predicted by the theoretical model you must convert the measured absolute positions into positions relative to the central peak.
Find in the plot the x-value corresponding to the central peak.
In our example x = 25.03 mm

- To convert the position values from absolute to relative enter the formula " A2-25,03 " into cell C2 and copy it into the subsequent cells then add the column header in cell C1
- To subtract the background light intensity from the measured values find in the plot the minimum value of light intensity (in our example it is y = 0.009).
- To perform the subtraction follow the same steps as above, with the difference that now you choose as output a new column (column D).

The resulting plot should be the following.



The simulation of the spectrum of a single slit wide a, placed at the distance D from the screen is obtained (see Theoretical Model) by plotting the function:

$$I(x) = Io\ (\sin y/y)2 \text{ with } y(x)= \pi a\ x / D\ l$$

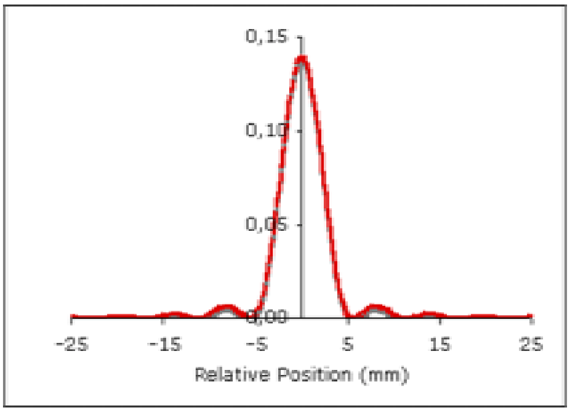
a = slit width
D = slit-screen distance
Io = central peak light intensity
l = light wavelength

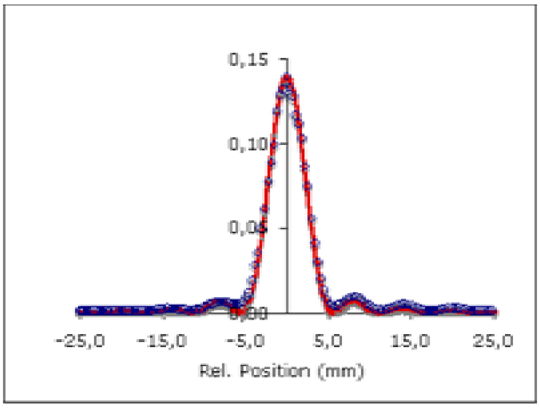
The values given to these parameters in our example are:
a = 80 µm
D = 700 mm
Io = 0.1392 (in order to normalize the peak to the measured value)
l = 0.645 µm

To build the simulated spectrum you first calculate in cell E2 the function y(x)= πax/lD with appropriate values of the parameters using the first value of x (cell A2) : enter the formula "=3.14*80*A2/(0.645*700)"
Then you copy it into the subsequent cells, then add the column header in cell E1
Then you calculate in cell F2 the function I(y)= I o [sin(y)/y]2: enter the formula "=0.1392*(SIN(E2)/E2)^2"
Then you copy it into the subsequent cells, then add the column header in cell F1

You will see that a warning appear: a division by zero gives no result in the cell corresponding to x =0. To overcome this problem (undefined value of the calculated function) you type in that cell the value of Io.

Finally choose plot for column C (in x axis) and F (y axis): you should obtain a plot like the following one.





To compare the experimental and simulated spectra you may display both on the same plot, as shown in the picture on the right.