Real Pendulum

www.lepla.eu
SOCRATES MINERVA Programme
99843-CP-1-2002-1-PL

The investigation is designed to provide information on the behavior of a pendulum, i.e. a small mass hanging from a string or a thin bar and free to oscillate on a plane.
The idea is to make a guided tour through pendulum motion, non restricted to small oscillations, comparing it to the motion of a simple harmonic oscillator and observing how its features are affected by the choice of the various system components.

Content:

- Theoretical model
- Apparatus setup
- Data acquisition
- Data sample
- Data analysis
- Evaluation form
- Teachers guide

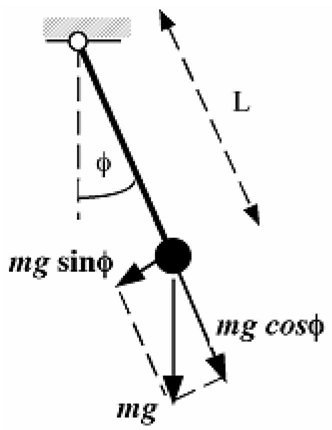
Theoretical model

Let us make some predictions on the characteristics of the motion of a pendulum made of a mass ***m*** attached to a thin bar of length ***L*** hanging from a fixed stand.The restoring torque is due to gravity and to pivot reaction and depends on the oscillation amplitude ϕ

$F = - m\, g \sin\phi \qquad M = - L\, m\, g \sin\phi$

(point mass approximation: L≈ pivot to center of mass distance)

$a_{ang} = M/I = -(g/L)\sin\phi$ with $I = m\,L^2$



Note that according to this equation torque is ***not*** proportional to angular displacement !
Only for small oscillations (**sin** $\phi \approx \phi$) the following *approximation* is valid:

$a_{ang} = -(g/L)\,\phi = -\omega^2\phi$

with angular frequency $\omega = \sqrt{g/L}$

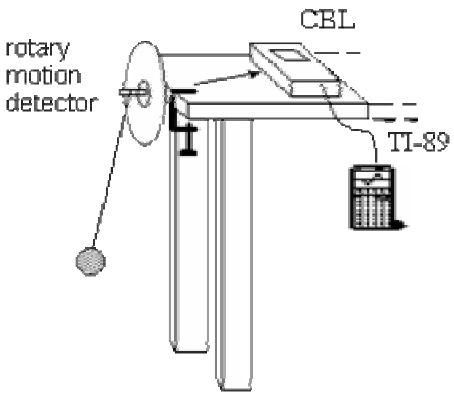
and period $T = 2\pi/\omega = 2\pi \cdot \sqrt{L/g}$ = ***constant*** (isochronism)

The equation is satisfied by the following solution

$\phi = \phi_0 \cos(\omega t+\beta) \qquad v_{ang} = \phi_0 \sin(\omega t+\beta) \qquad a_{ang} = \phi_0\, \omega^2 \cos(\omega t+\beta)$,

where ϕ_0 is the amplitude and β is the phase that depends from the position of the mass at t=0

Apparatus setup



To carry out the suggested experiments the following equipment is needed:

- Rotary motion sensor
- Graphic calculator (TI-83 or TI-89 /TI92+/Voyage200)
- CBL or LabPro interface

Balls of different weight that can be fixed at different position on a stick attached to rotary motion sensor

Qualitative observations
Displace the pendulum from equilibrium and let it oscillate.
Is the motion “regular”?
What seems to remain constant from one oscillation to the other?
Try to move the ball along the bar and let the pendulum go. Does the length seem to affect the motion?
Change the ball with a heavier one. Does the weight seem to affect the motion?

Data acquisition

To set up the data acquisition system and select the data acquisition mode, you can act as follows:

- Connect the rotary motion detector to the data acquisition system, and launch the application SCIENCE
- on MAIN MENU select 1: SET UP PROBES;
- on menu NUMBER OF PROBES select 1: ONE;
- on the third menu SELECT PROBE, select ROT.ENCODER .
- To measure displacements from equilibrium position on MAIN MENU select 5: ZERO PROBES, and, on “choose channel” select 1.
- At the request to press the START / STOP button on CBL2 verify first that the pendulum is steady at rest. (This procedure allows the angle to be measured in a reference frame whose origin is in the equilibrium position)
- On MAIN MENU select 2: COLLECT DATA.
- Choose SAMPLE TIME= 0.1 s and NUMBER OF SAMPLES = 100.

Your system is now ready for collecting data
Gently displace the pendulum up to a little less than 180° and leave it free. Start the data acquisition by pressing ENTER. You will obtain three plots for angular position, velocity and acceleration as functions of time.
A sample of data and graphs to be compared with those you obtained can be loaded from DATA SAMPLE
If you are satisfied with the collected data you should save them by selecting SAVE from the MAIN MENU, choosing the choice “else” and then typing a name of 3 letters at least.

Data Sample

Experimental data are downloadable in various formats from CD or web site:

- TI-89/92: pendulum.89C
- Excel : pendulum.xls
- Graphical Analysis: pendulum.ga3
- Logger Pro: pendulum.xmbl

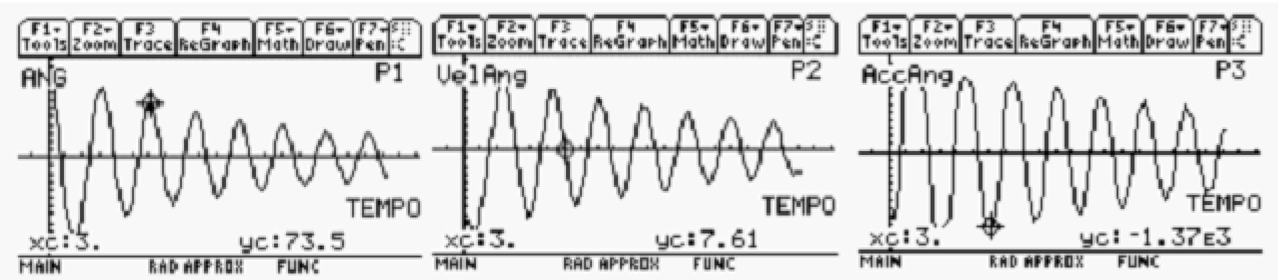
Data Analysis

Many information can be drawn from the collected data. Below you find the list of suggested types of analysis you can fiind in the next pages or reach with a click on CD or on web site.

- Plots analysis
- Period vs. amplitude
- Small oscillations
- Acceleration vs. position
- Velocity vs. position

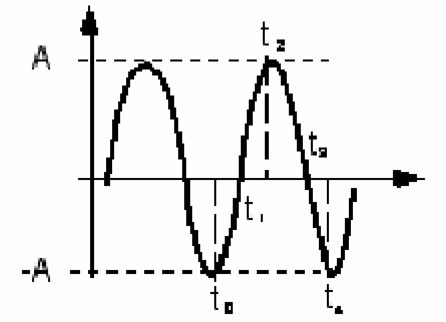
The analysis here documented was performed within the SCIENCE software, but alternative programs may be used (DataMate, Data/Matrix Editor, Graphical Analysis, Excel, LoggerPro….).

I. Plot Analysis



A few questions to be answered while analyzing the graphs can help you recognize the main features of the phenomenon.

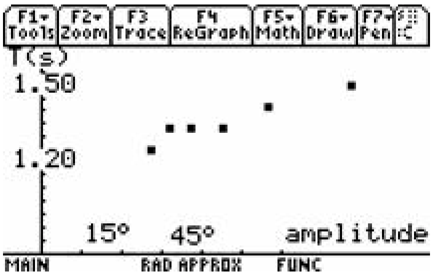
1. How large is the oscillation amplitude A (i..e the maximum elongation)?
2. How long is the time interval after which the motion repeats virtually identical?



3. Find the time intervals when the angular acceleration is positive and those when it is negative.
4. When is the absolute value of the angular velocity maximum? And when is it minimum?
5. Do you notice anything strange in the velocity and acceleration plots, at large amplitude? Can you find a reason for this?

II - Period vs. amplitude

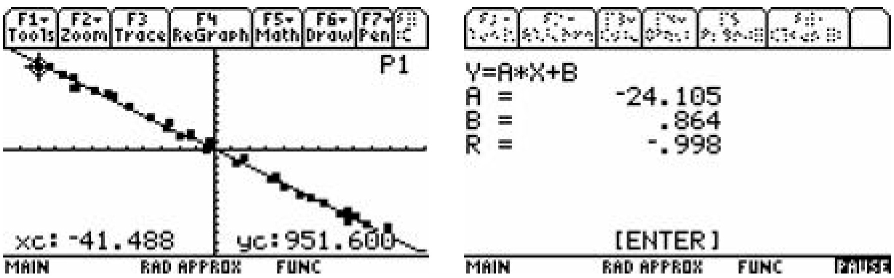
Look at the plots of angular position, velocity and acceleration versus time.
The shape of the graphs seems to be sinusoidal and using the cursor you can estimate the value of the period of oscillation for different values of the amplitude. What do you notice? Is the value constant or not?
If you try to plot the period versus the angular potion, you will probably find a plot similar to the following, showing that period increases with amplitude.



You can compare the experimental values of the period you found with that expected from theory (in the experiment documented here: T_0 = 1.27 s with L = 0.40 m).
Which values are compatible with the theoretical one?
To which experimental conditions do they correspond?

III - Small oscillations

When the amplitude is very little the pendulum behaves almost as an harmonic oscillator.
We notice that the period of the pendulum decreases with the amplitude towards a limit value that is the value theoretically expected in the small oscillations approximation.
We see that the plot of angular acceleration versus position is almost linear for small displacements from equilibrium.
But how “small” should small oscillations be?
In our experiment perhaps around 40° amplitude is already a good approximation for small oscillations.
Let’s try to check this by plotting only a selection of the original data (maximum amplitude around 40°).

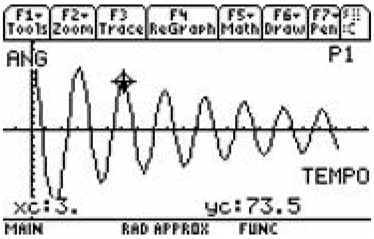


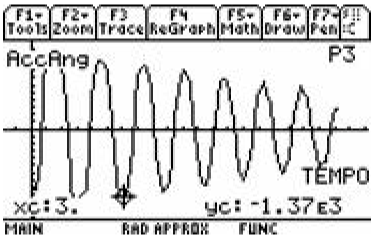
Angular acceleration vs. position plot, linear regression

We notice that in the selected data the relation a(t)=-const*x(t) holds between acceleration and position.
Which is the physical meaning of the constant?
Try a dynamical analysis, referring to the theoretical model of simple pendulum.

Alternative methods to evaluate the period.a_{ang}= - (g/ L) f = - w^2 f
1. From the plot of acceleration versus position we find a=–$\mathbf{w}^2$x and from the slope $\mathbf{w}^2$x we obtain T=2p/**w** (with the above data: T = 1.28 s)
2. A "sinusoidal" interpolation of the position versus time plot Y=A(sin (BX)+C)+D gives the best fitting value of the angular frequency B=**w** and therefore of the period T=2p/**w**

IV - Acceleration vs. position

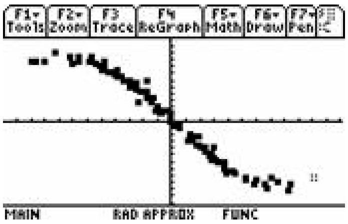




Look at the two plots of angular position and acceleration vs. time. To compare them it may be useful to put them one beneath the other.

You can notice that the period is the same, but phases are in opposition. We may expect that a linear relationship holds between angular acceleration and position.
To check this hypothesis we plot acceleration vs.position

What kind of graph do you expect?
Is our expectation correct?

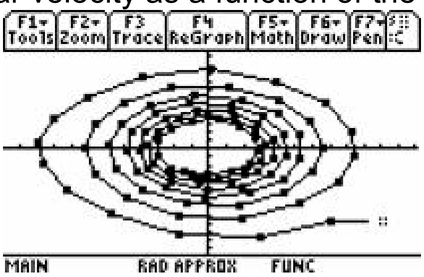


Plot of acceleration vs.position

You may notice that you cannot draw a straight line trough your data, except for the central part of the graph. To what phase of the motion does the central part correspond?

V - Velocity vs. position

A less usual graphical representation of the pendulum motion may be obtained by plotting angular velocity as a function of the angular position.



Why is the plot a spiraling curve?
Which ideal situation would give for this plot an ellipsis?

Teacher's Guide

References to related material:

- Pecori, B., Torzo, G., Pezzi, G., Foà, O., Rambelli, A., Rafanelli, M. Et Rizzo, M.R.: L'utilisation d'acquisitions avec des installations portables dans l'enseignement de la physique, *Bulletin de Physicien* **12** (2001)1833-1844
- B. Pecori, G.Torzo and A. Sconza,:Harmonic and anharmonic oscillations investigated by using a microcomputer-based Atwood's machine, *Am. J. Phys.*, **67**, (1999) 228-23
- G.Torzo, B.Pecori: Physics of the seesaw, *The Physics Teacher*, **39**, 491-495 (2001)

Answers to the questions in the Plot analysis
A few questions to be answered while analyzing the graphs can help you recognize the main features of the phenomenon.

1. *How large is the oscillation amplitude A (i..e the maximum elongation)?*
The ordinate of maxima or minima in the "distance" plot.
2. *How long is the time interval after which the motion repeats virtually identical?*
The time interval between two peaks in the plot
3. *Find the time intervals when the angular acceleration is positive and those when it is negative.*
It is positive from t_0 and t_1 and negative from t_1 to t_3
4. *When is the absolute value of the angular velocity maximum? And when is it minimum?*
The absolute value of the ang. velocity is maximum when the body crosses the oscillation midpoint (zero displacement). It is minimum (zero) at maximum elongation (angle).
5. *Do you notice anything strange in the velocity and acceleration plots, at large amplitude? Can you find a reason for this?*
Because the restoring force is F = - mg sinf, for small f values we may approximate sinf by f, (expecting a sinusoidal behavior) and for f close to p/2 we may approximate sinf by cos(p/2-f)= const.